§ 4B Simple and Compound Interest.

Definition: Principal - Money in an account on which interest is paid.

Simple Interest

This type of interest is calculated using only the starting principal in an account.

Ex: You invest \$1000 in an account paying 5% simple interest yearly. Determine the account principal, interest after years 1 and 2, and the total amount in the account after 5 years.

Solution: The principal is just \$1,000.

After 1 year, you earn interest of

$$\$1,000 \times .05 = \$50$$

In year 2, you earn interest of

$$\$1,000 \times .05 = \$50$$

This is true for simple interest: in one "period" (in this case a period is one year) you always earn the same interest. So, after n periods, you'll have: principal x interest rate x n = Int.

Here, \$1000 x 5 x .05 = \$250 in interest over 5 years. At the end of 5 years, your account has \$1000 + \$250 = \$1250.

Simple interest is an example of linear growth.

Compound Interest

With this type of interest, you earn interest on the original principal as well as any previous interest.

There are several types of compound interest:

1. Yearly compounding
2. Compounding n times per year
3. Compounding continuously

Definition: APR means Annual Percentage Rate

1 Yearly Compounding

Ex: You deposit \$1000 in a bank account paying an APR of 5% compounded yearly. Determine interest earned after year 1 and again after year 2. Compare this with the last simple interest example.

Solution: The principal is \$1000.

After year 1 you have in interest:

$$\$1000 \times .05 = \$50 \text{ interest}$$

This is identical to what you earned with simple interest after year 1.

After year 2, however, you earn

($\$1,000 + \$50) \times .05 = \$52.50$ in interest.

This is $2.50 more than the $50 you earned in simple interest during year 2.

Why ($1000 + $50)? Because, with compound interest you earn interest on previous interest as well as the principal.

Ex: In the previous example, how much money will you have in your account after 2 years?

Solution: The original principal is $1000. You earn $50 interest after year 1 and $52.50 after year 2. So you have in your account

$$\$1,000 + \$50 + \$52.50 = \$1,102.50$$

after 2 years.

In general, the balance after 1 year is:

$$\text{Principal} \times (1 + \text{APR})^1$$

In general, the balance after 2 years is:

$$\text{Principal} \times (1 + \text{APR})^2$$

If we continue this pattern, we get the Compound Interest Formula

Compound Interest Formula

Let Y = # of years

P = principal

A = Accumulated money in account

APR = Annual percentage rate

$$A = P \times (1 + APR)^{Y}$$

Ex: You deposit \$1,000 in an account paying APR = 10% compounded yearly. How much money will be in your account after 2 years? How much of this is interest?

Solution: Using the Compound Interest Formula gives how much, A, is in the account in 2 years

$$A = \$1000\,(1 + .1)^2$$
$$= 1000\,(1.1)^2$$
$$= \$1210$$

Since the principal was \$1000, the interest earned was \$1210 − \$1000 = \$210

2.) Compounding n times per year

By modifying the Compound interest Formula, we get the Compounding n times per year Compound Interest Formula.

Formula for Compounding n times per Year

Let y, APR, A, and P be as in the compound interest formula.

Let n be the number of compounding periods yearly.

$$A = P \times \left(1 + \frac{APR}{n}\right)^{ny}$$

Ex: You deposit \$500 in an account paying 5% APR compounded quarterly. How much will you have in the account after 10 years?

Solution: We seek A. Using the formula:

$$A = \$500 \times \left(1 + \frac{.05}{4}\right)^{4 \cdot 10}$$

$$= \$500 \times (1.0125)^{40}$$

$$= \$821.81$$

NOTE: $n = 4$ since we are compounding quarterly. IE, we compound every 3 months. However, n is NOT the number of months. It is the number of compounding periods.

Ex: You deposit \$1500 in an account paying 3% interest compounded monthly. How much will you have after 3 years?

Solution: Here, there are 12 compounding periods each year. So, $n = 12$. Then,

$$A = \$1500\left(1 + \frac{.03}{12}\right)^{12 \cdot 3}$$
$$= \$1641.08$$

3. Compounding Continuously

Continuous compounding gives you the best interest payment for a given APR. In general, the more times you compound at a given APR per year, the more interest you earn.

Compounding continuously formula:
Let A, P, Y, and APR be as in the Compounding n times per year formula.
Then,
$$A = P \times e^{(APR \times Y)}$$
where e is the exponential.

Ex: You deposit \$1000 in an account paying 8% APR compounded continuously. How much will you have in the account after 5 years?

Solution: $A = \$1000 \times e^{.08 \times 5}$
$= \$1491.82$

More examples of interest computations

Ex: You set up an account at a bank. This account pays 5% interest compounded every 2 months. If you deposit \$1000 in this account, how much will you have in 15 years?

Solution: This is compound interest. Here, $n = 6$. So, we seek A:

$$A = \$1000\left(1 + \frac{.05}{6}\right)^{15 \cdot 6}$$

$$= \$1000\,(2.11043) = \$2110.43$$

Ex: If you deposit \$200 in an account paying simple interest yearly, how much interest will you earn in 10 years? Take the interest rate as 2%.

Solution: This is simple interest. So,

$\$200 \times .02 = \$4$ interest per year.

over 10 years, you will earn

$10 \times \$4 = \$40$

Ex: You deposit \$100 in an account paying compound interest monthly at APR 2%. How much money will you have in the account after 3 years? How much is principal? How much is interest?

Solution: We seek first the amount in the account after 3 years. This is compound interest with $n = 12$ since we compound each month. So,

$$A = \$100\left(1 + \frac{.02}{12}\right)^{12 \cdot 3}$$
$$= \$106.18 \text{ after 3 years.}$$

The principal is the original \$100

The interest is \$106.18 - \$100 = \$6.18

Ex: You borrow \$1000 on a credit card charging an interest rate of 2% monthly. If interest is compounded twice per year, how much will you owe in 3 years assuming you make no payments or charges to the card?

Solution: We need A, the accumulated amount in the account (in this case debt).

$$A = \$1000 \times (1 + 6(.02))^{3 \cdot 2}$$
$$= \$1000 \times (1.12)^6$$
$$= \$1,973.82$$

Here, since we are charged 2% per month, that means we are charged $2\% \times 6 = 6(.02)$, which is the same as $(2\% \times 12)/2$, where $2\% \times 12 = 24\%$ is the APR.

The Annual Percentage Yield (APY)

Definition: The APY is the actual percentage by which a balance increases each year. If interest is compounded annually, then APY = APR. If interest is compounded more than once per year, then APR < APY.

Ex: You earn $82.50 interest on a principal of $1000 over 1 year. What is the APY?

Solution: APY is a relative change in the account size. So,

$$\text{Relative Change} = \frac{\text{Absolute change}}{\text{Original value}}$$

Here, the "original value" is the starting principal. What is the absolute change? Well, Accumulated Amount − Principal equals the interest earned. This is the absolute change in the account, the interest.

Thus,

$$\text{Relative Change} = \frac{\$82.50}{\$1000} = 8.25\%$$

So, the APY = 8.25%